

# Effect of mass asymmetry on the mass dependence of balance energy.

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**Abstract.** We demonstrate the role of the mass asymmetry on the balance energy ( $E_{bal}$ ) by studying asymmetric reactions throughout the periodic table and over entire colliding geometry. Our results, which are almost independent of the system size and as well as of the colliding geometries indicate a sizeable effect of the asymmetry of the reaction on the balance energy.

## 1. Introduction

Reaction dynamics has always captured a central place in nuclear physics mainly due to the wider domain of physics it caters. Right from the low energy (where fusion-fission takes place) to the ultra-relativistic energies (where sub-nucleonic degrees of freedom are dominant) a large number of new phenomena has been observed/predicted. The reaction dynamics in heavy-ion collisions at intermediate energies has been used extensively during the last three decades to understand the nature of nuclear matter at extreme conditions of temperature and density and the nature of nuclear equation of state (EoS) [1, 2, 3, 4, 5, 6, 7]. The phenomena that are mainly observed in this energy range are mainly multifragmentation, collective transverse flow, particle production etc.

In the search of nuclear equation of state as well as of nuclear interactions and forces, collective flow has been found to be of immense importance [1, 2, 3, 4, 5, 6, 7]. Among collective flow, transverse in-plane flow enjoys special status. During the last two decades, much emphasis has been put on the study of collective flow [1, 2, 3, 4, 5, 6, 7]. Lots of experiments have been performed and number of theoretical attempts have also been employed to explain and understand these observations. As reported by [8, 9] for the first time and later on by many others, collective flow is negative at low incident energies whereas it is positive at a reasonable higher incident energies. At a particular incident energy, however, a transition occurs. This transition energy is also known as energy of vanishing flow or balance energy ( $E_{bal}$ ). This balance energy ( $E_{bal}$ ) has been subjected to intensive theoretical calculations using variety of equations of state as well as nucleon-nucleon cross-sections [1, 2, 3, 4, 5, 6, 7, 8, 9]. This also includes the mass-dependence of  $E_{bal}$  which have been reproduced successfully by various theoretical models [1, 3, 4, 5, 6, 7].

Interestingly, most of these studies take symmetric or nearly symmetric reactions into account. Recently, FOPI group studied the flow for the asymmetric reaction of  $^{40}\text{Ca} + ^{197}\text{Au}$  [10, 11]. They noted that the flow in the asymmetric collisions is a key observable for investigating the reaction dynamics. In contrast to symmetric collisions, where center of mass is one of the nucleus, this

quantity is not known *a priori* in asymmetric nuclei experimentally. Later on, FOPI conducted experiment on  $^{58}\text{Ni}$  and  $^{208}\text{Pb}$  [10, 11]. In other class of studies, the total mass of the system was kept fixed as 96 units whereas charge was varied [12]. Theoretically, recently Kaur and Kumar [13] conducted a complete study of the multifragmentation by varying the asymmetry of the colliding nuclei. Asymmetry parameter  $\eta$  is defined as  $(A_T - A_P)/(A_T + A_P)$ ; where  $A_T$  and  $A_P$  are the masses of the target and projectile, respectively. All these attempts point towards a need for the systematic study of the disappearance of flow for asymmetric colliding nuclei. Further as noted, asymmetry of a reaction plays dramatic role in heavy-ion collisions [14, 15, 16, 17]. This happens because excitation energy in symmetric colliding nuclei leads to larger compression while asymmetric reactions lack the compression since large part of the excitation energy is in the form of thermal energy.

Note that some isolated studies with asymmetric nuclei are already done in the literature where the reactions of  $^{20}\text{Ne}+^{12}\text{C}$ ,  $^{20}\text{Ne}+^{27}\text{Al}$ ,  $^{20}\text{Ne}+^{63}\text{Cu}$ ,  $^{58}\text{Ni}+^{12}\text{C}$ ,  $^{64}\text{Zn}+^{27}\text{Al}$ ,  $^1\text{H}+^{197}\text{Au}$ ,  $^{12}\text{C}+^{197}\text{Au}$ ,  $^{197}\text{Au}+^{12}\text{C}$ ,  $^{40}\text{Ar}+^{207}\text{Pb}$  etc are taken into account [10, 11, 18, 19, 20, 21, 22, 23, 24, 25, 26]. Interestingly, none of the studies focus on the  $E_{bal}$  for asymmetric colliding nuclei. To address this, we here present a systematic study of the  $E_{bal}$  as a function of asymmetry of the colliding nuclei. While total mass of the reactions remain fixed, the asymmetry is varied by transferring the neutrons/protons from one nucleus to other. The QMD model used for the present analysis is explained briefly in the section 2. Results and discussion are presented in section 3 followed by summary in section 4.

## 2. Description of the model

The quantum molecular dynamics model (QMD) simulates the reaction on an event by event basis [2]. This is based on a molecular dynamics picture where nucleons interact via two and three-body interactions. The explicit two and three-body interactions preserves the fluctuations and correlations which are important for  $N$ -body phenomenon such as multifragmentation [2].

In the QMD model, the (successfully) initialized nuclei are boosted towards each other with proper center-of mass velocity using relativistic kinematics. Here each nucleon  $\alpha$  is represented by a Gaussian wave packet with a width of  $\sqrt{L}$  centered around the mean position  $\vec{r}_\alpha(t)$  and mean momentum  $\vec{p}_\alpha(t)$  [2]:

$$\phi_\alpha(\vec{r}, \vec{p}, t) = \frac{1}{(2\pi L)^{3/4}} e^{[-\{\vec{r}-\vec{r}_\alpha(t)\}^2/4L]} e^{[i\vec{p}_\alpha(t)\cdot\vec{r}/\hbar]}. \quad (1)$$

The Wigner distribution of a system with  $A_T + A_P$  nucleons is given by

$$f(\vec{r}, \vec{p}, t) = \sum_{\alpha=1}^{A_T+A_P} \frac{1}{(\pi\hbar)^3} e^{[-\{\vec{r}-\vec{r}_\alpha(t)\}^2/2L]} e^{[-\{\vec{p}-\vec{p}_\alpha(t)\}^2 2L/\hbar^2]}, \quad (2)$$

with  $L = 1.08 \text{ fm}^2$ .

The center of each Gaussian (in the coordinate and momentum space) is chosen by the Monte Carlo procedure. The momentum of nucleons (in each nucleus) is chosen between zero and local Fermi momentum [=  $\sqrt{2m_\alpha V_\alpha(\vec{r})}$ ;  $V_\alpha(\vec{r})$  is the potential energy of nucleon  $\alpha$ ]. Naturally, one has to take care that the nuclei, thus generated, have right binding energy and proper root mean square radii.

The centroid of each wave packet is propagated using the classical equation of motion [2]:

$$\frac{d\vec{r}_\alpha}{dt} = \frac{dH}{d\vec{p}_\alpha}, \quad (3)$$

$$\frac{d\vec{p}_\alpha}{dt} = -\frac{dH}{d\vec{r}_\alpha}, \quad (4)$$

where the Hamiltonian is given by

$$H = \sum_{\alpha} \frac{\vec{p}_{\alpha}^2}{2m_{\alpha}} + V^{tot}. \quad (5)$$

Our total interaction potential  $V^{tot}$  reads as [2]

$$V^{tot} = V^{Loc} + V^{Yuk} + V^{Coul} + V^{MDI}, \quad (6)$$

with

$$V^{Loc} = t_1 \delta(\vec{r}_{\alpha} - \vec{r}_{\beta}) + t_2 \delta(\vec{r}_{\alpha} - \vec{r}_{\beta}) \delta(\vec{r}_{\alpha} - \vec{r}_{\gamma}), \quad (7)$$

$$V^{Yuk} = t_3 e^{-|\vec{r}_{\alpha} - \vec{r}_{\beta}|/m} / (|\vec{r}_{\alpha} - \vec{r}_{\beta}|/m), \quad (8)$$

with  $m = 1.5$  fm and  $t_3 = -6.66$  MeV.

The static (local) Skyrme interaction can further be parametrized as:

$$U^{Loc} = \alpha \left( \frac{\rho}{\rho_o} \right) + \beta \left( \frac{\rho}{\rho_o} \right)^{\gamma}. \quad (9)$$

Here  $\alpha, \beta$  and  $\gamma$  are the parameters that define equation of state. The momentum dependent interaction is obtained by parameterizing the momentum dependence of the real part of the optical potential. The final form of the potential reads as

$$U^{MDI} \approx t_4 \ln^2 [t_5 (\vec{p}_{\alpha} - \vec{p}_{\beta})^2 + 1] \delta(\vec{r}_{\alpha} - \vec{r}_{\beta}). \quad (10)$$

Here  $t_4 = 1.57$  MeV and  $t_5 = 5 \times 10^{-4} \text{ MeV}^{-2}$ . A parameterized form of the local plus momentum dependent interaction (MDI) potential (at zero temperature) is given by

$$U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^{\gamma} + \delta \ln^2 [\epsilon (\rho/\rho_0)^{2/3} + 1] \rho/\rho_0. \quad (11)$$

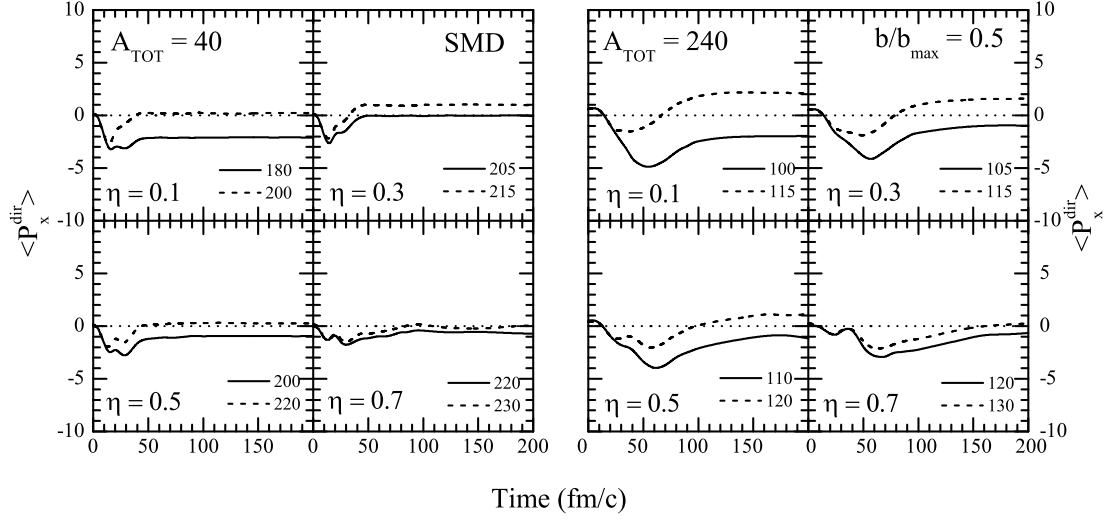
The parameters  $\alpha, \beta$ , and  $\gamma$  in above Eq. (11) must be readjusted in the presence of momentum dependent interactions so as to reproduce the ground state properties of the nuclear matter. The set of parameters corresponding to different equations of state can be found in Ref. [2].

### 3. Results and Discussion

For the present study, we simulated various reactions for 1000-5000 events in the incident energy range between 90 and 500 MeV/nucleon. In particular, we simulated the reactions of  $^{17}\text{O} + ^{23}\text{Na}$  ( $\eta = 0.1$ ),  $^{14}\text{N} + ^{26}\text{Mg}$  ( $\eta = 0.3$ ),  $^{10}\text{B} + ^{30}\text{Si}$  ( $\eta = 0.5$ ), and  $^6\text{Li} + ^{34}\text{S}$  ( $\eta = 0.7$ ) for total mass ( $A_{TOT}$ ) = 40,  $^{36}\text{Ar} + ^{44}\text{Ca}$  ( $\eta = 0.1$ ),  $^{28}\text{Si} + ^{52}\text{Cr}$  ( $\eta = 0.3$ ),  $^{20}\text{Ne} + ^{60}\text{Ni}$  ( $\eta = 0.5$ ), and  $^{10}\text{B} + ^{70}\text{Ge}$  ( $\eta = 0.7$ ) for total mass ( $A_{TOT}$ ) = 80,  $^{70}\text{Ge} + ^{90}\text{Zr}$  ( $\eta = 0.1$ ),  $^{54}\text{Fe} + ^{106}\text{Cd}$  ( $\eta = 0.3$ ),  $^{40}\text{Ca} + ^{120}\text{Te}$  ( $\eta = 0.5$ ), and  $^{24}\text{Mg} + ^{136}\text{Ce}$  ( $\eta = 0.7$ ) for total mass ( $A_{TOT}$ ) = 160, and  $^{108}\text{Cd} + ^{132}\text{Ba}$  ( $\eta = 0.1$ ),  $^{84}\text{Sr} + ^{156}\text{Dy}$  ( $\eta = 0.3$ ),  $^{60}\text{Ni} + ^{180}\text{W}$  ( $\eta = 0.5$ ), and  $^{36}\text{Ar} + ^{204}\text{Pb}$  ( $\eta = 0.7$ ) for total mass ( $A_{TOT}$ ) = 240. The present study is for peripheral collisions (i.e.  $b/b_{max} = 0.5$ ). Note that in some cases, slight variation can be seen for charges. The charges are chosen in a way so that colliding nuclei are stable nuclides. A momentum dependent soft equation of state with standard energy dependent cugnon cross-section (labeled as SMD) is used for the present calculations.

The balance energy ( $E_{bal}$ ) is calculated using the *directed transverse momentum*  $< P_x^{dir} >$ , which is defined as:

$$\langle P_x^{dir} \rangle = \frac{1}{A} \sum_i \text{sign}\{Y(i)\} \mathbf{p}_x(i), \quad (12)$$

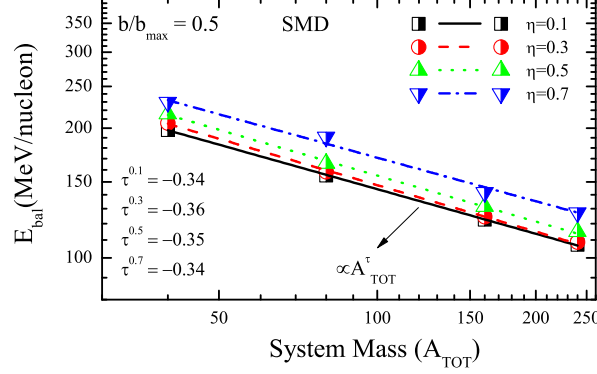


**Figure 1.** The time evolution of the directed transverse flow  $\langle P_x^{dir} \rangle$  as a function of time for different mass asymmetries. Columns 1 and 2 from left are for system mass ( $A_{TOT}$ ) = 40, whereas columns 3 and 4 are for system mass ( $A_{TOT}$ ) = 240. Here we display the results for peripheral collisions at different incident energies using SMD equation of state.

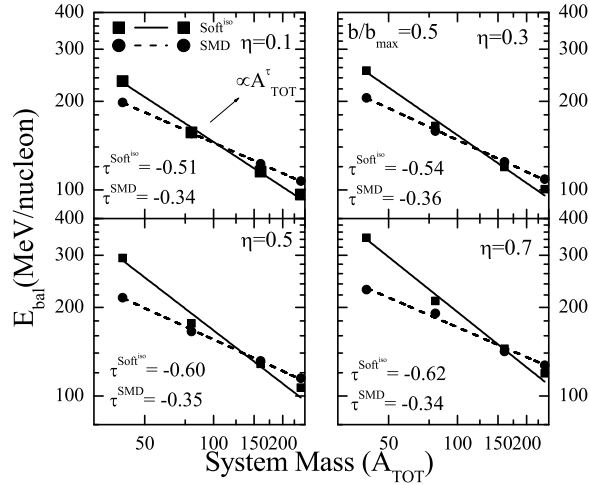
where  $Y(i)$  and  $\mathbf{p}_x(i)$  are the rapidity distribution and transverse momentum of  $i^{th}$  particle, respectively.

In Fig. 1, we display the time evolution of the directed transverse flow  $\langle P_x^{dir} \rangle$ , for various asymmetric reactions at different incident energies with system mass ( $A_{TOT}$ ) = 40 and 240 units. From the figure, it is clear that transverse flow is always negative during the initial phase of the reaction irrespective of incident energy, system mass, and asymmetry of the reaction. This shows that the interactions among nuclei are attractive during initial phase of the reaction. These interactions turn repulsive depending on the incident energy. The transverse in-plane flow in lighter colliding nuclei for all values of  $\eta$  saturates earlier compared to heavy colliding nuclei. A sharp transition from negative to positive flow is seen for lighter nuclei at lower asymmetry. For the heavier nuclei at all asymmetries; the transition is gradual. From the figure, one can see that nearly symmetric reactions ( $\eta = 0.1$  and  $0.3$ ) respond strongly to the change with incident energy compared to highly asymmetric reactions. The cause behind is that with the increase in the asymmetry of a reaction lesser binary collisions take place resulting in lesser density and therefore, less response occurs for variation in the collective flow. On the other hand, for nearly symmetric reactions, the binary collisions increases linearly with incident energy, therefore, huge difference can be seen.

It would be interesting to see how mass dependence character of the  $E_{bal}$  behaves at a fixed asymmetry. In Fig. 2, we display the  $E_{bal}$  verses combined system mass by keeping the asymmetry fixed. We notice that in all the cases, a perfect power law dependence (with power factor close to  $1/3$ ) can be seen for all asymmetries right from 0.1 to 0.7. The values of power factor are -0.34, -0.36, -0.35, and -0.34, respectively, for  $\eta = 0.1, 0.3, 0.5$ , and  $0.7$ . Strikingly, the results are similar to the one as shown recently by Chugh and Puri [27]. There it was predicted that the role of MDI for symmetric systems is more dominant at peripheral geometries and the value of power factor is nearly equal to -0.35. From the figure, it is clear that all points are



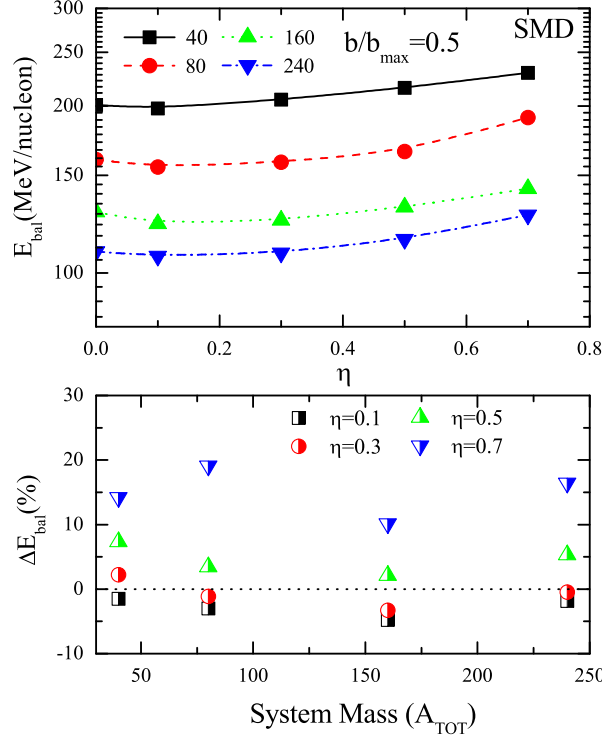
**Figure 2.** (Color Online) The  $E_{bal}$  as a function of system mass ( $A_{TOT}$ ) of reacting partners for SMD EoS and for peripheral collisions. The results for different asymmetries  $\eta = 0.1, 0.3, 0.5$ , and  $0.7$  are represented, respectively, by the half filled squares, circles, triangles and inverted triangles. Lines are power law fit  $\propto A_{TOT}^\tau$ .



**Figure 3.** The  $E_{bal}$  as a function of system mass. Solid squares(circles) are for  $\text{Soft}^{iso}$ (SMD) EoS. The top(bottom) left and right panels are for asymmetry parameter of  $0.1(0.5)$  and  $0.3(0.7)$  respectively. Lines are power law fit  $\propto A_{TOT}^\tau$ .

lying on the line, indicating that asymmetry plays a major role in the mass dependence. The variation in  $E_{bal}$  for  $\eta$  varying between  $0.1$  and  $0.7$  is  $\approx 32, 35, 19$ , and  $19$  for  $A_{TOT} = 40, 80, 160$ , and  $240$ , respectively. This explains why in the earlier mass dependence studies [1], though average behavior was a power law, individual  $E_{bal}$  were quite far from the average law [1, 3, 4]. There this happened because no control was made for the asymmetry of a colliding pair. Had all reactions been analyzed on a fixed asymmetry, one could have obtain perfect match with average power law.

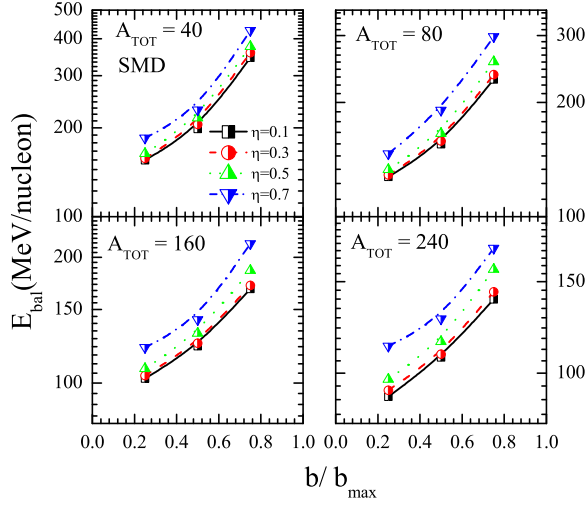
The effect of MDI on the  $E_{bal}$  is shown in Fig. 3, where we display the  $E_{bal}$  as a function of combined mass of the system with  $\text{Soft}^{iso}$  ( $\text{Soft}^{iso}$  represents the soft equation of state with isotropic standard energy dependent cugnon cross-section) and SMD EoS for  $\eta = 0.1, 0.3, 0.5$ ,



**Figure 4.** (Color Online)  $E_{bal}$  as a function of asymmetry parameter  $\eta$  is displayed in the upper panel, whereas the percentage difference  $\Delta E_{bal}(\%)$  as a function of system mass of reacting partners is shown in the lower panel. SMD EoS and an reduced impact parameter of 0.5 is used in both cases. The results for the systems having total mass  $A_{TOT}$  of 40, 80, 160, and 240 are represented, respectively, by the solid squares, circles, triangles and inverted triangles. Lines are to guide the eye. The results of the percentage difference for different asymmetries  $\eta = 0.1, 0.3, 0.5$ , and  $0.7$  are represented, respectively, by the half filled squares, circles, triangles and inverted triangles.

and  $0.7$ . Lines represent the power law fitting and different symbols are explained in the caption of the figure. It is clear from the figure that as  $\eta$  increases, the suppression of  $E_{bal}$  for the lighter systems with MDI also increases while the increase in  $E_{bal}$  for heavier nuclei remains almost same. Since transverse flow decreases with increase in asymmetry due to the decrease in nn collisions and Coulomb repulsions, therefore, the effect of the repulsive nature of MDI increases with increase in asymmetry mainly for the lighter masses. With the increase of asymmetry of the reaction, the role of MDI on the mass dependence of  $E_{bal}$  is similar to its role for the symmetric reactions at semi-peripheral and peripheral geometries.

In Fig. 4, we display  $E_{bal}$  as a function of  $\eta$  for a fixed mass equal to 40, 80, 160, and 240 units (top panel). In agreement with all previous studies,  $E_{bal}$  decreases with increase in the mass of the system. This decrease has been attributed to the increasing role of Coulomb forces in heavier colliding nuclei. As discussed in earlier figures, a sizeable influence can be seen towards  $E_{bal}$  with variation of the asymmetry of a reaction. For lighter masses, the effect of the variation of  $\eta$  can result about 35 MeV change in the  $E_{bal}$ . As noted in absolute terms, lighter nuclei are more affected compared to heavier ones. Overall, one sees that the effect of the asymmetry of a reaction is not at all negligible. It can have sizeable effect which goes as power law with power factor close to  $1/3$ . We also display the percentage difference  $\Delta E_{bal}(\%)$  defined as  $\Delta E_{bal}(\%) = ((E_{bal}^{\eta \neq 0} - E_{bal}^{\eta = 0}) / E_{bal}^{\eta = 0}) \times 100$  as function of system mass (bottom panel). Very interestingly, we



**Figure 5.** (Color Online) The  $E_{bal}$  as a function of reduced impact parameter. The top(bottom) left and right panels are for  $A_{TOT} = 40(80)$  and  $160(240)$  respectively. The results for different asymmetries  $\eta = 0.1, 0.3, 0.5$ , and  $0.7$  are represented, respectively, by the half filled squares, circles, triangles and inverted triangles. Lines are to guide the eye.

see that the effect of the asymmetry variation is almost uniform throughout the periodic table. In other words, asymmetry of a reaction can play significant role in  $E_{bal}$  and the deviation from the mean line can be eliminated if proper care is taken for the asymmetry of colliding nuclei.

In Fig. 5, we display the  $E_{bal}$  as a function of the impact parameter for different asymmetries. A well known trend i.e. increase in the  $E_{bal}$  with impact parameter can be seen for all ranges of system mass. Further, as demonstrated by many authors, the impact parameter variation is less effected in the presence of MDI. The striking result is that the effect of mass asymmetry variation is almost independent of the impact parameter.

#### 4. Summary

Using the quantum molecular dynamics model, we presented a detailed study of the balance energy with reference to mass asymmetry. Almost independent of the system mass as well as impact parameter, an uniform effect of the mass asymmetry can be seen at the energy of vanishing flow. We find that for large asymmetries, ( $\eta = 0.7$ ), the effect of asymmetry can be 15% with MDI and in the absence it can be 40%. This also explain the deviation in the individual  $E_{bal}$  from the mean values as reported earlier.

#### 5. Acknowledgments

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